AN EXPORT MODEL FOR THE BELGIAN INDUSTRY

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AN EXPORT MODEL FOR THE BELGIAN INDUSTRY*

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A two-equation model for the export volume and the export price of the Belgian industry is specified as a convex combination of demand and supply determinants and estimated using Bayesian inference methods. The results indicate that, in the medium run, industrial exports are mainly explained by the behaviour of price taking suppliers.

1. Introduction

Export activity is very well known to be vital for the so-called 'small open' Belgian economy, and therefore its modelling is also vital for the good performance of macroeconometric models of that economy.

However, traditional export volume equations of Belgian econometric models have been demand equations while at the same time the export price equations were highly dependent on foreign competitors' prices.[1] This resulted in a passive behaviour of Belgian exports. It was usually difficult to discover significant export price elasticities since the demand behaviour of Belgium's customers would depend on a very stable competitive export price ratio. The analysis of exports finally ended up with the projection of the Belgian export market share following some constant elasticity on world trade and a trend induced from the recent past.

The approach presented below starts from explicit bilateral trade demand and supply curves on specific markets and develops a model representing convex combinations of demand and supply determinants for the export volume and export price (section 2). An empirical evaluation for the Belgian industrial sector is presented in section 3, using Bayesian inference methods presented in an appendix.

The model presented in this paper has been introduced by the authors in the international block of SERENA, a large macroeconometric model of the

*This work was initiated during the construction of the SERENA model for the Belgian Planning Office, and pursued at CORE. The first author is partially supported by the 'Projet d'Action Concertée: Applications de la Théorie des Décisions Economiques' of the government of Belgium. These various supports are herewith gratefully acknowledged. The authors thank J.H. Drèze and a referee for helpful comments and suggestions.

[1]See, for example, Thys-Clement et al. (1973).

Belgian economy disaggregated into nine sectors and three regions. Industry is the major sector and the equation for industrial exports is of crucial importance for the model since they represent around 75% of total exports and half of total industrial production.

SERENA has been built by the second author with the Belgian Planning Office, in view of preparing the 1981–1985 plan.

2. A model of determination of export volume and export price

For a bilateral single commodity market between an exporting country i and an importing country j, a simple[2] supply function can be represented by

$$\ln QX_{ji} = \mu_{0ji} + \mu_{ji} \ln QX_{Si} + \pi_i \ln (PX_{ji}/PX_i), \qquad \mu_{ji} > 0, \quad \pi_i > 0, \tag{2.1}$$

and a corresponding demand equation by

$$\ln QM_{ij} = \beta_{0ij} + \beta_{ij} \ln QM_{Dj} + \psi_j \ln (PM_{ij}/PM_j), \qquad b_{ij} > 0, \quad \psi_j < 0. \tag{2.2}$$

The following definitions are used for the commodity considered:

QX_{ji} = volume of exports of country i towards country j;
QX_{Si} = total export supply determinants of country i;
PX_{ji} = bilateral export price;
PX_i = average (over countries of destination) export price of country i;
QM_{ij} = volume of imports of country j from country i;
QM_{Dj} = total import demand determinants of country j;
PM_{ij} = bilateral import price;
PM_j = average (over countries of origin) import price of country j.

An aggregate model is derived under the assumption of *perfect flexibility of the bilateral trade price*. The bilateral market is thus cleared by the bilateral price adjustment. Using the equilibrium conditions

$$\ln PX_{ji} = \ln PM_{ij} + c,^{3}$$

$$\ln QX_{ji} = \ln QM_{ij},$$

the export and import allocation mechanisms (2.1) and (2.2) can be solved to express the equilibrium price and quantity in function of the supply and demand determinants QX_{Si} and QM_{Dj}, and of the average prices PX_i and PM_j.

Defining $\alpha_{ij} = \pi_i/(\pi_i - \psi_j)$ $(0 < \alpha_{ij} < 1$ since $\pi_i > 0$ and $\psi_j < 0)$, the solution is written as

$$\ln PX_{ji} = c' + \alpha_{ij} \ln PX_i + (1 - \alpha_{ij}) \ln PM_j$$
$$+ (1/(\pi_i - \psi_j))(\beta_{ij} \ln QM_{Dj} - \mu_{ji} \ln QX_{Si}), \tag{2.3}$$

$$\ln QX_{ji} = c'' + \alpha_{ij}\beta_{ij} \ln QM_{Dj} + (1 - \alpha_{ij})\mu_{ij} \ln QX_{Si}$$
$$+ \psi_j \alpha_{ij} \ln (PX_i/PM_j). \tag{2.4}$$

The traded quantity appears therefore as a log linear convex combination of the demand and supply determinants, while the trade price appears as the same combination of the average export and import prices. The two remaining relative quantity and relative price terms can be seen as long run equilibrium conditions related to the adding-up of world trade. When they are ignored in the short run, or over short periods available for estimation, the interpretation of (2.3) and (2.4) can be lifted from the particular cases where either π_i or ψ_j is close to zero.

In the case $\pi_i = 0$ (case I) the traded volume results from the export supply function of country i, while the trade price PM_j has to be interpreted as the competitive export price (a weighted sum of the export prices of the competitors of country i on the market of country j).

When the exporter does not have a predominant position on the market of country j for the commodity considered, he behaves usually as a price taker. The bilateral export price is then likely to be proportional to the competitors' export price. Therefore the exporter fixes the level of exports in function of the supply determinants. Markets of this type can be referred to as *demand markets*, since one could use the same equations under the assumption of rigid bilateral prices and persistent excess demand, i.e., when at the given export prices exports are constrained by supply.

In the case $\psi_j = 0$ (case II) the traded volume results from the import demand equation of country j, while the trade price is the average export price of country i. This price will be assumed to result from an average or marginal cost price calculation with a 'normal' constant markup.

On this kind of market the exporter behaves as a price maker, fixing the export price in function of production costs, not necessarily because he has a predominant position on that market, but also when a profit squeeze threatens his viability. Since he fixes the export price, the exporter must then accept the level of exports determined by the demand side of the market. These markets can be referred to as *supply markets*, since the equations are

[2]More refined versions of this model have been presented by Barten and d'Alcantara (1977).
[3]c is a constant of proportionality between PX_{ji} and PM_{ij}, due e.g. to import duties.

also compatible with rigid bilateral prices, related to production costs, and persistent underutilisation of capacities.

In short, demand markets are represented by an export price given by the competitors' price ($\ln PX_{ji} = c + \ln PXW_i$) and by an export volume given by a supply equation ($\ln QX_{ji} = c' + \mu_{ji} \ln QX_{Si}$); supply markets are represented by an export price related to a production cost index ($\ln PX_{ji} = k + \ln PB_i$) and by an export volume given by a demand equation ($\ln QX_{ji} = k' + \beta_{ij} \ln QM_{Dj}$).

The total export volume equation of country j is obtained as a weighted aggregate over j of all the bilateral volume eqs. (2.4). The corresponding export price equation results from (2.3). Using a constant α_i the system of two equations can be represented at the aggregate level as

$$\ln PX_i = c' + \alpha_i \ln PB_i + (1-\alpha_i) \ln PXW_i, \tag{2.5}$$

$$\ln QX_i = c'' + \alpha_i \ln QX_{Di} + (1-\alpha_i) \ln QX_{Si}, \tag{2.6}$$

where PB_i is a production cost index and PXW_i is a competitors' export price index. The constant α_i is interpreted as the proportion of supply markets, i.e., of demand determinants in the volume of exports and of the production cost in the export price.

This result is exact when PX_i is in the nature of a geometric index — say $PX_i = PX_{iI}^{1-\alpha_i} PX_{iII}^{\alpha_i}$, with I denoting demand markets and II denoting supply markets. It is hoped that formulae (2.5) and (2.6) do not contain serious specification errors under the prevailing definition of PX_i.

3. Empirical results for the Belgian industry (1965–1976)

The export model of the Belgian industrial sector in SERENA is specified as the simultaneous equations model (2.5)–(2.6), where the unobservable variables $\ln QX_D$ and $\ln QX_S$[4] are taken as log linear relationships in their arguments or lagged values of them.

The demand function depends on a weighted sum of imports by Belgium's customers ($\ln QMW$), on the rate of change of this variable ($\Delta \ln QMW$) and on the ratio of the Belgian export prices to the competitors' prices [$\ln(PX/PXW)$].[5]

The supply function depends on the potential output ($\ln QP$) and on a proxy for the profitability of exports, namely the ratio of the export price to the production cost [$\ln(PX/PB)$].

[4]The index i can now be deleted.

[5]The competitors' price PXW is built as a Belgian franc converted weighted sum of the export prices of the competitors, using as weights the share of each competitor on the international export market.

The model is finally specified and parametrised as follows:

$$\ln(PX/PXW) = \alpha \ln(PB/PXW) + u_p, \tag{3.1}$$

$$\begin{aligned}\ln QX &= \beta_0 + \alpha(\eta_1 \ln QMW + \eta_2 \ln(PX/PXW) \\ &\quad + \eta_3 \ln(PX_{-1}/PXW_{-1}) + \eta_4 \Delta \ln QMW) \\ &\quad + (1-\alpha)(\gamma_1 \ln QP_{-1} + \gamma_2 \ln(PX_{-1}/PB_{-1})) + u_q, \qquad (3.2) \\ &\equiv \beta_0 + \beta_1 \ln QMW + \beta_2 \ln(PX/PXW) \\ &\quad + \beta_3 \ln(PX_{-1}/PXW_{-1}) + \beta_4 \Delta \ln QMW \\ &\quad + \beta_5 \ln QP_{-1} + \beta_6 \ln(PX_{-1}/PB_{-1}) + u_q. \qquad (3.2')\end{aligned}$$

The dynamic specification of (3.2) is the result of some preliminary search.

The reparametrisation (3.2′) of the volume equation has been used for three reasons:

(a) as α cannot be 0 or 1 ($0<\alpha<1$), the estimations of (3.1)–(3.2) and of (3.1)–(3.2′) yield the same results, e.g. the estimate of α is the same, and the estimate of η_1 is equal to the estimate of β_1 divided by that of α, using the maximum likelihood method;[6]

(b) to the extent that the elasticities of QX with respect to the different exogenous variables (QMW, PXW, QP, PB) are the parameters of interest, the parametrisation, (3.1)–(3.2′) is more convenient;

(c) (3.1)–(3.2′) is linear in its parameters, while (3.1)–(3.2) is not.

Posterior expectations and standard deviations obtained with different prior measures along the lines explained in the appendix are presented in table 1.

Column (1) corresponds to the non-informative prior measures presented in the appendix, i.e., a prior proportional to

$$|\Sigma|^{-\frac{3}{2}}[1+(\beta-\beta_0)'(\beta-\beta_0)]^{-4} I_{(0.05,\,0.95)}(\alpha), \tag{3.3}$$

where $\beta_0 = (\beta_{00}\beta_{10}\beta_{20}\beta_{30}\beta_{40}\beta_{50}\beta_{60})'$ is the vector of modes of the prior

[6]Using Bayesian methods, the posterior density of α will also be the same; that of η_1 can be obtained either directly if the parameterisation (3.1)–(3.2) is used, or through the change of variable $\eta_1 = \beta_1/\alpha$ if (3.1)–(3.2′) is used; these densities must also be identical (within numerical accuracy). Of course, the posterior expectation $E(\eta_1)$ will differ from $E(\beta_1)/E(\alpha)$.

Table 1

β	(1)[a]	(2)[b]	(3)[c]	(4)[d]
β_1	0.98 (0.11)	0.62 (0.12)	0.49 (0.10)	1.04 (0.07)
β_2	−0.55 (0.83)	−4.13 (6.54)	−0.77 (1.09)	0.08 (0.32)
β_3	−1.06 (0.46)	−2.18 (1.18)	−1.88 (0.78)	−0.78 (0.36)
β_4	0.22 (0.19)	0.73 (0.31)	0.91 (0.32)	0.09 (0.13)
β_5	−0.11 (0.24)	0.83 (0.10)	1.08 (0.18)	−0.31 (0.17)
β_6	0.53 (0.35)	1.07 (0.84)	1.40 (0.45)	0.27 (0.24)
α	0.18 (0.09)	0.17 (0.10)	0.21 (0.07)	0.16 (0.09)
ξ	−0.72 (0.40)	−1.84 (1.03)	−1.68 (0.46)	−0.34

[a]Diffuse prior proportional to $|\Sigma|^{-\frac{3}{2}} I_{(0.05,0.95)}(\alpha) \times [1+\sum_{i=0}^{6}(\beta_i-\beta_{i0})^2]^{-4}$, where the β_{i0} are given by (3.4).

[b]Diffuse price proportional to $|\Sigma|^{-\frac{3}{2}} I_{(0.05,0.95)}(\alpha)$, and exact constraint $\beta_5=1-\alpha$.

[c]Prior proportional to $|\Sigma|^{-\frac{3}{2}}(\alpha-0.05)^2(0.95-\alpha \times I_{(0.05,0.95)}(\alpha)\ f_t(\beta_1,\beta_2,\beta_3,\beta_5,\beta_6)$, where $f_t(\cdot)$ is the Student prior with expectations and covariance matrix (3.3)–(3.4) and 30 degrees of freedom.

[d]FIML estimates; the asymptotic estimate of the standard deviation of ξ has not been computed.

Cauchy on β, with value

$$\beta_0=(0 \quad 1 \quad -0.75 \quad -0.75 \quad 0 \quad 1.33 \quad 1.5). \tag{3.4}$$

These modal values are the prior expectations (except for β_0 and β_4) of the Student prior used below and will be explained when this prior is described.

The prior used to obtain the results of column (2) is (3.3) without the kernel involving β; in counterpart a deterministic constraint is introduced: The parameter γ_1 of the supply function in (3.2) is constrained to be equal to 1, which means that $\beta_5=1-\alpha$ is the coefficient of $\ln QP_{-1}$ in the volume equation (3.2′).

This constraint has been introduced tentatively because the posterior expectation of β_5 in column (1) reveals an unlikely negative elasticity of exports with respect to the potential output. This unreasonable estimate is due to the collinearity between the variables $\ln QMW$, $\Delta \ln QMW$ and $\ln QP_{-1}$. The linear combination $\beta_1+\beta_5-\beta_4$ is estimated as 0.63 (with a relatively small standard deviation of 0.06), and constraining any of the parameters β_1, β_5 or β_4 to zero does not change the point estimate of that linear combination.[7]

It can be seen in column (2) that the effect of the constraint $\beta_5=1-\alpha$ is rather strong on all the parameters except α. In particular, price elasticities become more sizable, as shown by the elasticity ξ of exports with respect to the production cost,

$$\xi=\alpha(\beta_2+\beta_3)-(1-\alpha)\beta_6,$$

of which the first two posterior moments are presented in table 1. This is also the opposite of the elasticity of QX with respect to PXW or to the exchange rate of the Belgian franc with respect to the dollar (because PXW is equal to PXW in dollars times this exchange rate). An order of magnitude of ξ between -1 and -2 seems more plausible than the value -0.72 in column (1).

In order to relax the deterministic constraint on β_5 (i.e., $\beta_5=1-\alpha$)[8] and at the same time to offset somehow the multicollinearity of the data, prior information has been introduced on this parameter. As it represents the elasticity of exports with respect to the potential output, it is assigned a prior expectation greater than unity, to reflect the tendency of the small open economy of Belgium to be more and more integrated in the international trade; such a value implies an increase of the share of exports in the potential output. The value chosen is 1.33 with a standard deviation of 0.16, resulting in a 0.95 prior probability interval without values below 1 (using a normal approximation).

As the price taking behaviour is thought to be the most usual one among Belgian exporters, because of their fairly typical position of 'atoms' on external markets, the share α of supply markets is constrained to be a priori definitely less than 0.5 with a rather high probability. A beta prior density on the interval (0.05, 0.95) is used, with parameters 3 and 6, implying a mode of 0.275, an expectation of 0.33 and a standard deviation of 0.135. The prior probability that α is less than 0.5 is then equal to 0.86.

Prior information on the partial elasticities β_2; β_3 and β_6 has been elicited to reflect the prior opinion that the long run elasticity ξ of exports with respect to the production cost is less than -1, without considering this value as unlikely. The prior moments chosen are

$$\mathrm{E}(\xi)=-1.5, \qquad \mathrm{V}(\xi)=0.25,$$

[7]These computations were made using the asymptotic covariance matrix given by FIML estimation. FIML point estimates and estimated standard deviations are reported in column (4) of table 1.

[8]This constraint implies that β_5 is between 0.05 and 0.95 a priori with probability 1, in contradiction with our prior beliefs.

so that the value -1 is only one standard deviation away from the expectation.

Under the assumption of prior independence between $(\beta_2, \beta_3, \beta_6)$ and α, $E(\xi) = E(\alpha)E(\xi_1) - [1 - E(\alpha)]E(\beta_6)$, where $\xi_1 = \beta_2 + \beta_3$. Sharing $E(\xi)$ between the two terms (i.e., the effects of the demand and of the supply) in the proportions $E(\alpha)(=0.33)$ and $1 - E(\alpha)$, and assuming that $E(\beta_2) = E(\beta_3)$ give

$$E(\beta_2) = E(\beta_3) = -0.75, \qquad E(\beta_6) = 1.5.$$

Under the additional assumption of prior independence between (β_2, β_3) and β_6,[9]

$$\begin{aligned} V(\xi) &= E_\alpha[V(\xi \mid \alpha)] + V_\alpha[E(\xi \mid \alpha)] \\ &= E(\alpha^2)V(\xi_1) + [1 + E(\alpha^2) - 2E(\alpha)]V(\beta_6) \\ &\quad + V(\alpha)[E^2(\xi_1) + E^2(\beta_6) + 2E(\xi_1)E(\beta_6)] \\ &= 0.13V(\xi_1) + 0.47V(\beta_6), \end{aligned}$$

because the factor multiplying $V(\alpha)$ is equal to 0. Sharing $V(\xi)$ between the two terms on a 50% basis yields

$$V(\xi_1) = 0.98, \qquad V(\beta_6) = 0.27.$$

Assuming finally that $V(\beta_2) = V(\beta_3)$ and computing $\mathrm{cov}(\beta_2, \beta_3)$ from a correlation coefficient of -0.5 to allow some substitution between the effects of the two parameters, one obtains

$$V(\beta_2) = V(\beta_3) = 0.98, \qquad \mathrm{cov}(\beta_2, \beta_3) = -0.49.$$

To reflect the idea of a constant market share of Belgian exports in the long run, the elasticity β_1 of exports with respect to the foreign demand is assigned a prior expectation equal to 1 with a standard deviation of 0.25.

A Student prior density on $(\beta_1\, \beta_2\, \beta_3\, \beta_5\, \beta_6)$ is used to represent the information described. It is given 30 degrees of freedom to verify the normality approximation. Its expectation vector and covariance matrix are, respectively,

$$(1 \quad -0.75 \quad -0.75 \quad 1.33 \quad 1.5), \tag{3.5}$$

[9]More generally, prior independence is assumed between the parameters of the demand and of the supply, since these functions represent independent behaviours of different agents, the price taker and the price maker exporters.

$$\begin{pmatrix} 0.06 & & & & \\ 0 & 0.98 & & & \\ 0 & -0.49 & 0.98 & & \\ 0 & 0 & 0 & 0.03 & \\ 0 & 0 & 0 & 0 & 0.27 \end{pmatrix} \tag{3.6}$$

The results with these prior densities on β and α are shown in column (3) of table 1. They reveal clearly that the prior information on β_5 dominates the sample information. This prior has the direct effect of increasing the posterior expectation of β_5 — compare columns (1) and (3) — but also the indirect effect of decreasing that of β_1, as the correlation between β_1 and β_5 is strongly negative [-0.92 in column (1), -0.72 in column (3)]. The decreased effect of $\ln QMW$, via β_1, is however compensated by the increased effect of $\Delta \ln QMW$, via β_4.

The prior information on the price elasticities has the expected result to push the total elasticity ξ of exports with respect to the production cost towards a reasonable value -1.68, with as 95% posterior probability interval $(-2.6, -0.76)$; the precision of this statement does not seem to be either excessive or too low.

Finally α, the proportion of 'price maker demand' determinants is estimated to be rather low, with a stable value of around 20% in the different runs; this confirms the intuitive knowledge that Belgian exporters are mainly price takers.

4. Conclusions

The model presented in this paper allows to take account of two kinds of export behaviour, though the available data base remains aggregated. Empirical results obtained for the Belgian industry show that the 'price taking supplier' behaviour accounts for around 80% of the determination of exports. A policy implication is that exports could be developed through the creation of production capacities whose technology permits to sell at a competing price with sufficient profitability.

Appendix: Bayesian analysis of the model

The model presented in section 3 can be compactly written as

$$\begin{aligned} p &= x\alpha + u_p \quad \text{(price equation)}, \\ q_\alpha &= Z\beta + u_q \quad \text{(quantity equation)}, \end{aligned} \tag{A.1}$$

where $q_\alpha = \alpha q_1 + q_2$; p, x, q_1 and q_2 are T vectors, and Z is a $T \times k$ matrix, of observations. For example, in the original model (3.1)–(3.2):

$$p = \ln(PX/PXW), \qquad x = \ln(PB/PXW), \qquad q_1 = 0, \qquad q_2 = \ln QX,$$

$$Z = (i \ln QMW \ln(PX/PXW) \ln(PX_{-1}/PXW_{-1})$$

$$\Delta \ln QMW \ln QP_{-1} \ln(PX_{-1}/PB_{-1})).$$

In the same model where $\gamma_1 = 1$, q_1 becomes $\ln QP_{-1}$ and $q_2 = \ln QX - \ln QP_{-1}$, while Z is the matrix above, without the sixth column. u_p and u_q are T vectors of unobservable disturbances and β is a k vector of parameters.

Under the usual hypothesis that the probability distribution of the $T \times 2$ matrix $(u_p\, u_q)$ is a matrix-normal with parameters $(0\,0)$ and $\Sigma \otimes I_T$ (where Σ is a PDS matrix of order 2), the *data density* is

$$D(p, q_\alpha \mid \Sigma, \beta, \alpha) \propto |\Sigma|^{-\frac{1}{2}T} \exp -\tfrac{1}{2} \mathrm{tr}\, \Sigma^{-1} K, \tag{A.2}$$

where

$$K = \begin{bmatrix} (p - x\alpha)'(p - x\alpha) & (p - x\alpha)'(q_\alpha - Z\beta) \\ (q_\alpha - Z\beta)'(p - x\alpha) & (q_\alpha - Z\beta)'(q_\alpha - Z\beta) \end{bmatrix}. \tag{A.3}$$

The *prior density* of Σ, β and α has been factorised as

$$D(\Sigma) D(\beta) D(\alpha), \tag{A.4}$$

where

$$D(\Sigma) \propto |\Sigma|^{-\frac{1}{2}(\nu + 3)}, \tag{A.5}$$

i.e., a non-informative prior measure where the parameter ν can be set to 0 in application of Jeffreys' invariance principle or to the number n of predetermined variables of the model in application of Drèze's invariance principle. Further,

$$D(\beta) \propto [1 + (\beta - \beta_0)' M_0 (\beta - \beta_0)]^{-\frac{1}{2}(\nu_0 + k)}, \tag{A.6}$$

i.e. a Student density with parameters β_0, M_0 and ν_0. $D(\alpha)$ can be chosen freely, for example as beta or uniform (to remain non-informative) densities on some interval.

The parameter ν in (A.5) is chosen to be 0 for two reasons:

(1) in view of the short sample period ($T = 11$), the choice $\nu = n = 7$ would result in much lower posterior standard deviations of β [as can be seen from (A.7)];

(2) with $D(\beta)$ Cauchy (i.e. $\nu_0 = 1$), this prior is in the class of the non-informative prior densities that are invariant with respect to the choice of the normalized coefficient in the quantity equation; this class has been defined by Drèze and Richard (1981, sect. 6.2) for the full information analysis of a model identified by exact a priori restrictions.

Integrating out Σ in the product of (A.2) by (A.5) and (A.6) yields as *posterior density* of β, conditional on α, a product form poly-t density [see Drèze (1977)],

$$D(\beta \mid \alpha, p, q_\alpha) \propto [1 + (\beta - \beta_0)' M_0 (\beta - \beta_0)]^{-\frac{1}{2}(\nu_0 + k)}$$

$$\times [s_* + (\beta - \beta_*)' M (\beta - \beta_*)]^{-\frac{1}{2}\nu_*}. \tag{A.7}$$

The second Student kernel is equal to $|K|^{-\frac{1}{2}\nu_*}$ which is a quadratic form in β. The parameters of this kernel are

$$M_* = Z'\Omega Z, \qquad \Omega = (p - x\alpha)'(p - x\alpha) I_T - (p - x\alpha)(p - x\alpha)',$$

$$\beta_* = M_*^{-1} Z'\Omega q_\alpha, \qquad s_* = q_\alpha' \Omega q_\alpha - \beta_*' M_* \beta_*, \qquad \nu_* = \nu + T.$$

N.B.: (A.7) is not valid if $\alpha = 0$. In this case, M_* is singular because its third diagonal element is zero, and the posterior density of β_3 is improper (not integrable), unless the prior is informative on it.

The density (A.7) can be marginalised with respect to α by one-dimensional numerical integration, using e.g. Gaussian rules. The posterior density of α needed for this step is obtained as

$$D(\alpha \mid p, q_2) \propto D(p, q_\alpha \mid \alpha) D(\alpha), \tag{A.8}$$

where the conditional predictive density $D(p, q_\alpha \mid \alpha)$ is proportional to the integral of the kernel of (A.7). The marginal *posterior density of* α is therefore obtained by numerical integration of (A.8).

More details can be found in Bauwens (1979) where alternative factorisations of the prior are presented.

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