# FACTOR DEMAND EXPLANATION
# IN THE COMET MODEL

by

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**CORE REPRINT No. 632**

**CENTER FOR OPERATIONS RESEARCH & ECONOMETRICS**

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# Factor demand explanation in the COMET model

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*The COMET model is an interconnected set of 13 econometric country models, one for each member country of the European Economic Community and for some other countries as well. Within each country model demand for production factors, including energy, is explained by a multi-output/multi-input national production structure resulting in an internally coherent subset of four input equations, which allows for reasonably realistic dynamics and involves only a limited number of unknown coefficients, which are internationally comparable. The estimates display considerable similarity across countries, rather weak price responses, but relatively strong substitution between labour and capital.*

*Keywords*: Macroeconometric model; Production structure; Estimation of price responses

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Final manuscript received 27 November 1983.

The COMET model to which the title refers is a recent version of a series of models of the European Economic Community (EEC) constructed for the staff of the European Commission. For an earlier version see Barten *et al* [1]. The major features of this model are presented in the next section.

The set of equations explaining inputs of production factors in COMET forms an internally coherent subsystem or module. The present formulation is inspired by the need to account adequately for possibilities of substitution and complementarity between capital services, labour, energy and other commodity imports. In this sense the module is similar to the KLEM factor demands approach of Hudson and Jorgenson [2] and to the economy-wide microeconometric modelling framework of Theil [4]. The module also explicitly allows for differences in effects in factor demand between different types of macroeconomic output, like private consumption, commodity exports. The multi-output/multi-input production function base is presented later.

Adjustment to a new equilibrium position is not instantaneous. It takes longer to have the desired capital stock installed than to have the right size and composition of the labour force, which, however, will also not be achieved within a year's period. Energy and other commodity imports will adjust more readily. Delays in adjustments cause spillovers onto other factor demands. The difference in delay between capital and labour introduces a cascade of such spillovers.

For a multicountry modelling project like COMET it is important to be able to compare easily estimation results across countries. Similar, although not equal, estimates inspire confidence in the procedure. Comparability helps identify outliers which then may be brought under control. The basic framework also contains constraints among the partial derivatives. Carrying these over to the coefficients to be estimated reduces their number and thus contributes to the efficiency of estimation. These parametrization issues are discussed later. Technological change is introduced, while other aspects of the transition of a set of theoretical relations to one of estimable equations are treated.

Actual estimation, its method and results for the 13 countries enumerated in Table 1 are presented too. The final section contains the major conclusions.

## The COMET context

The factor demand equations are a part of the COMET model. This model basically consists of a set of 13 interlinked models for national economies supplemented by some relation for five zones representing the rest of the world. Table 1 lists the countries for which a fully specified model is part of COMET, together with their symbols. The first nine countries are present members of the European Economic Community. In view of its relatively small size the Luxemburg economy is not represented by a model of its own. Portugal and Spain are candidate members and therefore already included in the overall model. The importance of the US and Japanese economies for the Western industrialized world has led to the inclusion of a national model for these two countries along the same lines as for the actual or prospective EEC members.

These national models comprise some 30 estimated equations each. Sets of bilateral trade equations and international price formation equations link these country models to each other and to five zones in which the rest of the world is classified. These five zones consist of the other members of the Organization for Economic Cooperation and Development (OECD) than those in Table 1; socialist countries; oil-exporting developing countries; 'fast' developing countries and 'other' developing countries. These zones are not described fully, but only by some 'feedback' relations linking imports to exports and exports prices to international prices. In principle, COMET covers the whole world. It represents a view of the world economy from an EEC vantage point.

The national models are dynamic models in the sense that in their estimated equations past values of endogenous (and exogenous) variables codetermine present ones. Some of the dynamics reflect the desire to capture the adjustment towards a normal level of the degree of utilization of capacity (DUC). Deviations from this normal level exert delayed corrective movements on capital demand and on prices. This feature represents the 'medium-term' nature of the COMET model, ie the mutual adjustment of supply of and demand for domestic production in the medium run.

The complexity of the full model requires that its components be simple enough to maintain intellectual command over the project not only for its construction but also for simulation and interpretation purposes. The (sub) models for 13 national economies have therefore been given an identical specification. The estimated equations are usually of the double-logarithmic type. The estimated coefficients are then elasticities which can be easily compared across countries. Differences between the various countries express themselves in differences in values of the coefficients and can be inspected for their plausibility.

Most of the equations have been estimated equation-by-equation by the method of least-squares with some exceptions like the factor demand modules described here. Frequently nonlinear estimation is applied because the derivation of the equations implies nonlinear combinations of interpretable coefficients. The database used starts at the earliest with data for 1953 and ends at the last with those of 1981. However, the same length of time series is not used for all countries, nor for all equations, due to restricted data availability. Major sources of data are the statistics published by the Statistical Office of the European

The authors gratefully acknowledge the assistance and comments of Dominique Warnier, Chris de Witte, Willem van Groenendaal and Denis de Crombrugghe, but accept sole responsibility for any errors.

Table 1. Country models in COMET.

| Country | Symbol |
|---|---|
| Federal Republic of Germany | DB |
| France | FR |
| Italy | IT |
| Netherlands | NL |
| Belgium | BE |
| UK | UK |
| Ireland | IR |
| Denmark | DK |
| Greece | HE |
| Portugal | PO |
| Spain | ES |
| USA | US |
| Japan | JA |

Communities (SOEC) and by the OECD, supplemented by data from the International Monetary Fund, the International Labour Office and the United Nations.

The national models contain equations for the various major categories of final demand. The purpose of the factor demand equations is to determine the inputs of hours of work, services by fixed assets other than housing, energy and non-energy commodity imports that these outputs require. In turn, these inputs will have effects on other variables. For example, commodity imports are allocated over countries and zones of origin where they will generate economic activity. For the overall working of the COMET model, the factor demand module is of vital importance. In view of the considerable changes over the last decades in technology, relative prices and composition of output a rather refined framework is needed which explicitly allows for the possibility of substition or complementarity among factors of production. It is natural to base this framework on a production function which allows such interactions.

## The derived factor demand system

It is a common procedure to treat a whole economy as if it were a single agent, characterized by optimizing behaviour. This procedure is not so much adopted for its realism but primarily because it supplies an interpretative framework and useful constraints for estimation and extrapolation. This approach is applied here to obtain a mutually consistent set of demand equations for inputs.

For convenience of notation let $f$ be the $p$-vector of outputs and let $c$ be the $q$-vector of inputs, with $p_c$ the $q$-vector of input prices. Next, let the production possibility frontier be characterized by the following multi-output/multi-input production function

$$g(f, c) = 0 \tag{1}$$

For a given output vector $f$, the country will use the inputs $c$ that minimize total costs: $p_c'c$. This optimal input vector will satisfy Equation (1) and the conditions

$$p_c = m\, g_c \tag{2}$$

where $m$ is a Lagrange multiplier and $g_c = \partial g/\partial c$. The conditions are equivalent to the traditional equality of marginal costs ($p_c$) to marginal revenue. Equations (1) and (2) constitute a system of $q + 1$ equations in the $q + 1$ unknowns, namely the elements of the vector $c$, the inputs, and $m$, the Lagrange multiplier. Under regularity conditions this system can be solved for these unknowns in the form of functions of $f$ and $p_c$.

To obtain such a solution production function (1) should be functionally specified, a far from straightforward matter. As long as one is interested primarily in locally valid responses of $c$ to shifts in $f$ and $p_c$ one can avoid a direct specification of Equation (1) and hence maintain some generality. This approach is taken here.

Let $df$ and $dp_c$ be shifts in the output vector $f$ and in the input price vector $p_c$, respectively. Equilibrium conditions (2) can then be written in differential form as

$$m\, G_{cc}dc + m\, G_{cf}\, df + g_c dm = dp_c \tag{3}$$

where $dc$ and $dm$ are 'optimal' changes in $c$ and $m$, respectively, while

$$G_{cc} = \frac{\partial g_c}{\partial c'} = \frac{\partial g}{\partial c\ \partial c'} = G'_{cc}$$

$$G_{cf} = \frac{\partial g_c}{\partial f'} = \frac{\partial g}{\partial c\ \partial f'}$$

The differential form of production function (1) reads

$$g'_f\, df + g'_c\, dc = 0 \tag{4}$$

with $g_f = \partial g/\partial f$. Combining Equations (3) and (4) results in the following matrix equation:

$$\begin{bmatrix} mG_{cc} & g_c \\ g'_c & 0 \end{bmatrix} \begin{bmatrix} dc \\ dm \end{bmatrix} = \begin{bmatrix} -mG_{cf} & I \\ -g'_f & 0 \end{bmatrix} \begin{bmatrix} df \\ dp_c \end{bmatrix} \tag{5}$$

Note that the matrix on the extreme left-hand side is a symmetric one. Its invertibility follows from regularity assumptions on the production function. Write this inverse as

$$\begin{bmatrix} Z & z \\ z' & \zeta \end{bmatrix} \tag{6}$$

which is also symmetric, implying symmetry of the $q \times q$ matrix $Z$. Then, premultiplying both sides of Equation (5) by matrix (6) results in the following expression for the equilibrium changes in $c$:

$$\begin{aligned} dc &= -(m\, ZG_{cf} + z\, g'_f)\, df + Z\, dp_c \\ &= B\, df + Z\, dp_c \end{aligned} \tag{7}$$

with $B = -(m\, ZG_{cf} + z\, g'_f)$

Result (7) is trivial unless $B$ and $Z$ satisfy certain special conditions. Indeed, there are such conditions. $Z$ is a symmetric matrix. Moreover, as can be shown, $p'_c Z = 0$, hence $Z\, p_c = 0$. The diagonal elements of $Z$ have to be negative. There is, however, only one constraint on $B$ : $p'_c B = -m\, g'_f$, which is not too useful. Note, however, that $B = \partial c/\partial f'$, ie the matrix of partial derivatives of inputs with respect to outputs, otherwise said the matrix of primary input contents for which input–output tables can supply empirical information. To avoid having to estimate these coefficients along with the others they can and have been calculated separately from input–output information. A summary documentation of these calculations is given in the Appendix.

One next defines

$$dq = B\, df \tag{8}$$

where $dq$ is a vector of 'synthetic' variables indicating the input responses for a given small change in outputs for the year for which $B$ has been measured and without any change in (relative) input prices. We can then write

$$dc = dq + Z\, dp_c \tag{9}$$

for the derived demand system. This version of the system puts the focus of econometric estimation on the matrix $Z$. It forms the basis for the rest of this article.

## Incomplete adjustment

The derived demand system (9) describes full adjustment of all inputs to a new equilibrium situation. It does not specify the time frame of this response. In the COMET project the yearly interval is the minimum time period. It is not realistic to assume that the production factor capital will have completed its adjustment within a year. In many countries labour adjusts with considerable inertia. One can also envisage future situations in which one of the inputs is made subject to quantity rationing. It is of use, therefore, to consider the case where one or more inputs do not adjust fully. It is clear that the optimal changes in the fully flexible inputs have to depend on the actual values of the more rigid inputs.

To proceed, let $(p_c^0, q^0)$ be the set of values of $(p_c, q)$ in the initial equilibrium situation and write $p_c^1 = p_c^0 + dp_c$, $q^1 = q^0 + dq$. According to Equation (9) full adjustment would mean:

$$c^{1*} - c^0 = q^1 - q^0 + Z(p_c^1 - p_c^0) \tag{10}$$

or in scalar notation, with $j = 1, \ldots, q$:

$$c_j^{1*} - c_j^0 = q_j^1 - q_j^0 + \textstyle\sum_{\ell=1}^{q} z_{j\ell}\,(p_{c\ell}^1 - p_{c\ell}^0) \tag{11}$$

Next, let $c_1$ be the constrained input, say capital or energy, which will not be adjusted completely but is in one way or the other fixed at the value $c_1^1 \neq c_1^{1*}$. Then, rational behaviour means cost minimization for given $c_1^1$ and, of course, as before for given $f$. The price of $c_1$ will play no role in the determination of the quantity of the other inputs.

In general terms one can write the optimal adjustment equations for inputs $j = 2, \ldots, q$, as

$$c_j^1 - c_j^0 = \bar{q}_j^1 - q_j^0 + \textstyle\sum_{\ell\neq 1} \bar{z}_{j\ell}\,(p_{c\ell}^1 - p_{c\ell}^0) + h_{j1}\,(c_1^1 - c_1^0) \tag{12}$$

where $\bar{}$ over $q_j^1$ and $z_{j\ell}$ indicates that these concepts are different from the corresponding ones of full adjustment. The last term represents most explicitly the lack of adjustment of $c_1$. Indeed, if $c_1^1$ were equal to $c_1^{1*}$ the conditional demands $c_j^1$ should be $c_j^{1*}$, $j > 1$. This property can be exploited to obtain expressions for $\bar{q}_j^l$, $\bar{z}_{j\ell}$ and $h_{j1}$ in terms of the elements of $q^1$ and of $z$. Substituting the right-hand side of Equation (11) with $j = 1$ for $c_1^1 - c_1^0$ in Equation (12) gives

$$\begin{aligned} c_j^1 - c_j^0 = \bar{q}_j^1 - q_j^0 + \textstyle\sum_{\ell\neq 1} \bar{z}_{j\ell}\,(p_{c\ell}^1 - p_{c\ell}^0) \\ + h_{j1}\,[q_1^1 - q_1^0 + \textstyle\sum_{\ell} z_{1\ell}\,(p_{c\ell}^1 - p_{c\ell}^0)] \end{aligned} \tag{13}$$

Equalizing the right-hand sides of Equations (11) and (13) then yields the following properties, with $j > 1$:

$$\bar{q}_j^1 - q_j^0 + h_{j1}\,(q_1^1 - q_1^0) = q_j^1 - q_j^0$$

$$\bar{z}_{j\ell} + h_{j1}\, z_{1\ell} = z_{j\ell}$$

$$h_{j1}\, z_{11} = z_{j1}$$

The last condition implies $h_{j1} = z_{j1}/z_{11}$ with $z_{11}$ being strictly negative. The second one allows one to write

$$\bar{z}_{j\ell} = z_{j\ell} - h_{j1}\, z_{1\ell} = z_{j\ell} - z_{j1}\, z_{1\ell}/z_{11}$$

Using also the first property one can rewrite Equation (12) as

$$c_j^1 - c_j^0 = q_j^1 - q_j^0 + \sum_\ell z_{j\ell}\,(p_{c\ell}^1 - p_{c\ell}^0)$$
$$+ (z_{j1}/z_{11})\,\{c_1^1 - c_1^0 - [(q_1^1 - q_1^0) + \sum_\ell z_{1\ell}\,(p_{c\ell}^1 - p_{c\ell}^0)]\}$$

The term in [ ] equals $c_1^{1*} - c_1^0$. Consequently one has

$$c_j^1 - c_j^0 = q_j^1 - q_j^0 + \sum_\ell z_{j\ell}\,(p_{c\ell}^1 - p_{c\ell}^0) + (z_{j1}/z_{11})\,(c_1^1 - c_1^{1*}) \tag{14}$$

or

$$c_j^1 - c_j^0 = c_j^{1*} - c_j^0 + (z_{j1}/z_{11})\,(c_1^1 - c_1^{1*}) \tag{15}$$

Some aspects of these results may be underlined. First of all, no new parameters are involved in Equation (14) as compared to Equation (11). Second, the last term with $(c_1^1 - c_1^{1*})$ represents the spillover effect of lack of adjustment of $c_1$. Remember that $z_{11}$ is negative. If inputs $j$ and 1 are substitutes and thus $z_{j1} > 0$, a shortage of $c_1$ will force $c_j^1$ above $c_j^{1*}$. In the case of complementarity such a shortage will leave $c_j^1$ below its $c_j^{1*}$ value. Third, $(c_1^1 - c_1^{1*})/z_{11}$ can be interpreted as the shadow price of the constraint. By simply adding $(c_1^1 - c_1^{1*})/z_{11}$ to $(p_{c1}^1 - p_{c1}^0)$ in Equation (11) the original system can be left formally intact.

This line of thought can be easily extended to more than one constrained input. The shadow price will then involve the inverse of a part of the $Z$ matrix. Here a different approach will be followed to extend the applicability of the system. This approach is based on a scenario of potentially less and less rigidly adjustable inputs. For example, capital adjusts slowly to its final equilibrium level, labour will move faster to its equilibrium level, the latter, however, being conditional on the actual available amount of capital. The other inputs adjust to the optimal values given the available amounts of capital and labour.

Returning to Equation (14) we will rewrite this expression as

$$\hat{c}_j^1 - c_j^0 = q_j^1 - q_j^0 + \sum_\ell z_{j\ell}\,(p_{c\ell}^1 - p_{c\ell}^0) + (z_{j1}/z_{11})\,(c_1^1 - c_1^{1*}) \tag{16}$$

for $j > 1$, where $\hat{}$ indicates that the adjustment is conditional on $c_1^1$. Next, assume that $c_2^1 \neq \hat{c}_2^1$ because of, say, lagged adjustment. A similar reasoning as before will show that for $j > 2$:

$$c_j^1 - c_j^0 = q_j^1 - q_j^0 + \sum_\ell z_{j\ell}\,(p_{c\ell}^1 - p_{c\ell}^0) + (z_{j1}/z_{11})\,(c_1^1 - c_1^{1*})$$
$$+ (z_{j2}/z_{22})\,(c_2^1 - \hat{c}_2^1) \tag{17}$$

or that

$$c_j^1 - c_j^0 = \hat{c}_j^1 - c_j^0 + (z_{j2}/z_{22})\,(c_2^1 - \hat{c}_2^1) \tag{18}$$

where it is to be noted that $c_2^1$ is taken in deviation from $\hat{c}_2^1$ as defined by Equation (16) and not in deviation from $c_2^{1*}$.

For practical purposes it is not very convenient to start off from an equilibrium input vector $c_j^0$. In fact, this is not necessary. Consider another $(q, p_c)$ combination, say $q^2, p_c^2$. The ensuing difference from a conceptual initial equilibrium situation $c_j^0$ can be written in a way analogous to Equation (17) as

$$c_j^2 - c_j^0 = q_j^2 - q_j^0 + \sum_\ell z_{j\ell} (p_{c\ell}^2 - p_{c\ell}^0) + (z_{j1}/z_{11}) (c_1^2 - c_1^{2*}) + (z_{j2}/z_{22}) (c_2^2 - \hat{c}_2^2) \quad (19)$$

with $\hat{c}_2^2$ defined analogously to Equation (16). Write $\Delta x = x^2 - x^1$, for $x = c,q,p$. Subtract from both sides of Equation (19) the corresponding sides of Equation (17) to obtain:

$$\Delta c_j = \Delta q_j + \sum_\ell z_{j\ell} \Delta p_{c\ell} + (z_{j1}/z_{11}) (\Delta c_1 - \Delta c_1^*) + (z_{j2}/z_{22}) (\Delta c_2 - \Delta \hat{c}_2) \qquad j > 2 \quad (20)$$

Here

$$\Delta c_1^* = \Delta q_1 + \sum_\ell z_{1\ell} \Delta p_{c\ell} \quad (21)$$

$$\Delta \hat{c}_2 = \Delta q_2 + \sum_\ell z_{2\ell} \Delta p_{c\ell} + (z_{21}/z_{11}) (\Delta c_1 - \Delta c_1^*) \quad (22)$$

The next step consists in specifying the delayed adjustments $\Delta c_1$ and $\Delta c_2$. For this purpose it is useful to associate the $(q^2, p_c^2)$ and $(q^1, p_c^1)$ situations with consecutive time positions and to write $(q_t, p_{ct})$ and $(q_{t-1}, p_{c,t-1})$, respectively. We will also write $\Delta c_{jt}$, $\Delta q_{jt}$, $\Delta p_{c\ell t}$ and so on, to make clear that these symbols indicate shifts over a fixed time period. The crucial assumption is now that $c_{1t}$ moves from $c_{1,t-1}$ towards $c_{1t}^*$ only *partially*:

$$\Delta c_{1t} = \kappa_1 (c_{1t}^* - c_{1,t-1}) \quad (23)$$

with $\kappa_1$ being a speed of adjustment parameter defined on the interval between zero and one. One next takes first differences on both sides of Equation (23):

$$\Delta c_{1t} = \kappa_1 \Delta c_{1t}^* + (1 - \kappa_1) \Delta c_{1,t-1} \quad (24)$$

Use of Equation (21) for $\Delta c_{1t}^*$ gives

$$\Delta c_{1t} = \kappa_1 (\Delta q_{1t} + \sum_\ell z_{1\ell t} \Delta p_{c\ell t}) + (1 - \kappa_1) \Delta c_{1,t-1} \quad (25)$$

with $\kappa_2$ being the speed of adjustment parameter for $c_2$ $(0 < \kappa_2 \leqslant 1)$ one obtains in an analogous way

$$\Delta c_{2t} = \kappa_2 (\Delta q_{2t} + \sum_\ell z_{2\ell t} \Delta p_{c\ell t}) + \kappa_2 (z_{21}/z_{11})_t (\Delta c_{1t} - \Delta c_{1t}^*) + (1 - \kappa_2) \Delta c_{2,t-1} \quad (26)$$

The consecutive partial adjustment scheme for $c_1$ and $c_2$ is specifically appropriate if $c_1$ is capital and $c_2$ labour or employment, which is the case here. With $c_1$ being capital there is a problem, however, since our observations refer to gross fixed investment $i_t$ and not to the capital stock $c_{1t}$. Assuming fixed proportional depreciation these two concepts are related by the *accumulation rule*

$$\Delta c_{1t} = i_t - \delta c_{1,t-1} \qquad 0 < \delta \leq 1 \tag{27}$$

with $\delta$ being the rate of depreciation. Taking first differences on both sides of Equation (27) and rearranging terms results in

$$\Delta i_t = \Delta c_{1t} - (1 - \delta)\,\Delta c_{1,t-1} \tag{28}$$

Next, we subtract from both sides of Equation (28) their lagged values multiplied by $(1 - \kappa_1)$:

$$\Delta i_t - (1 - \kappa_1)\,\Delta i_{t-1} = \Delta c_{1t} - (1 - \delta)\Delta c_{1,t-1} - (1 - \kappa_1)\,[\Delta c_{1,t-1} - (1 - \delta)\,\Delta c_{1,t-2}]$$

which can also be written as

$$\Delta i_t = [\Delta c_{1t} - (1 - \kappa_1)\,\Delta c_{1,t-1}] - (1 - \delta)\,[\Delta c_{1,t-1} - (1 - \kappa_1)\,\Delta c_{1,t-2}] + (1 - \kappa_1)\,\Delta i_{t-1}$$

Using Equation (25) the terms in square brackets can be replaced by terms in $\Delta q_1$ and $\Delta p_{c\ell}$, yielding after some rearrangement:

$$\Delta i_t = \kappa_1[\Delta q_{1t} - (1 - \delta)\,\Delta q_{1,t-1} + \textstyle\sum_\ell z_{1\ell t}\,\Delta p_{c\ell t} - (1 - \delta)\sum_\ell z_{1\ell,t-1}\,\Delta p_{c\ell,t-1}] + (1 - \kappa_1)\,\Delta i_{t-1} \tag{29}$$

which forms the basis for the estimation equation.

It is useful to point out the two types of dynamics underlying investment Equation (29). One stems from the partial adjustment scheme (23), the other from accumulation rule (27). The first of these is a behavioural inertia assumption, the other one is of a more technical kind. As it stands Equation (29) is a variant of the *flexible accelerator* explanation of investment.

The expressions for the other factors or inputs (20) and (26) contain a term in $\Delta c_{1t} - \Delta c^*_{1t}$. To eliminate $\Delta c^*_{1t}$ one can start off from Equation (24) which is transformed as

$$\Delta c_{1t} - \Delta c^*_{1t} = [(\kappa_1 - 1)/\kappa_1]\,(\Delta c_{1t} - \Delta c_{1,t-1}) \tag{30}$$

The coefficient $(\kappa_1 - 1)/\kappa_1$ in this expression is nonpositive. This relation should not be interpreted as a causal explanation. It rather states that accelerated growth in $c_1$, ie $\Delta c_{1t} > \Delta c_{1,t-1}$, is a symptom of an increasing discrepancy between actual $c_{1t}$ and optimal $c^*_{1t}$. As follows from Equation (28)

$$\Delta c_{1t} - \Delta c_{1,t-1} = \Delta i_t - \delta\Delta c_{1,t-1}$$

To simplify matters depreciation of the increase in capital stock is ignored. Consequently,

$$\Delta c_{1t} - \Delta c^*_{1t} = [(\kappa_1 - 1)/\kappa_1]\,\Delta i_t \tag{31}$$

is used in Equations (20) and (26).

To eliminate $\Delta\hat{c}_2$ from Equation (20) basically the same procedure is used, by specifying

$$\Delta c_{2t} - \Delta\hat{c}_{2t} = [(\kappa_2 - 1)/\kappa_2]\, \Delta^2 c_{2t} \qquad (32)$$

Since labour input $c_{2t}$ is directly observed $\Delta^2 c_{2t}$ is available and no further transformations are needed.

To summarize the results of this section: the demand equation for fixed capital is turned into one for investment – see Equation (29). The labour demand equation is written as

$$\Delta c_{2t} = \kappa_2 (\Delta q_2 t + \textstyle\sum_\ell z_{2\ell t}\, \Delta p_{c\ell t}) + \kappa_2(z_{21}/z_{11})_t\, [(\kappa_1 - 1)/\kappa_1]\, \Delta i_t + (1 - \kappa_2)\, \Delta c_{2,t-1} \qquad (33)$$

while the demand equations for the other inputs are

$$\Delta c_{jt} = \Delta q_{jt} + \textstyle\sum_\ell z_{j\ell t}\, \Delta p_{c\ell t} + (z_{j1}/z_{11})_t\, [(\kappa_1 - 1)/\kappa_1]\, \Delta i_t + (z_{j2}/z_{22})_t\, [(\kappa_2 - 1)/\kappa_2]\, \Delta^2 c_{2t}, \qquad j > 2 \qquad (34)$$

The next section will associate observable time series with these specifications and stipulate the constant coefficients to be estimated.

## Parameterization and related issues

The two preceding sections are of a predominantly theoretical nature. The next one reports on estimation. This section aims at linking these. This means for instance identifying the theoretical variables with those of the model and choosing a parameterization of the equations which allows convenient estimation and comparability across countries. The inclusion of supplementary explanatory variables is also dealt with here.

The inputs explained by the factor demand system are: (i) the number of hours worked per year in the non-government sector, *HEN*, indicated by $c_2$ in the preceding section; (ii) non-residential fixed investment, *IPO*, named $i_t$ before; (iii) energy, *QO*; and (iv) non-energy commodity imports, *MGO*. The other inputs considered in the Appendix – indirect taxes minus subsidies (*NIT*) and imports of services (*MSO*) – are explained outside the context of the factor demand system. *IPO*, *QO* and *MGO* are expressed in constant 1970 prices.

The corresponding factor prices are: (i) *PH*, the average hourly earnings in the private sector; (ii) *V*, capital user costs, which takes into account the purchasing price of the investment good, its interest charges and its depreciation;[1] (iii) *PQ*, the price index for energy; and (iv) *PMG*, the price index for commodity imports other than energy. All price indexes are equal to unity for 1970.

In the factor demand system derived in the preceding sections, the $z_{j\ell}$ are the price coefficients. Two useful constraints have been formulated for these. The first is the *symmetry property*: $z_{j\ell} = z_{\ell j}$. The second states that $\Sigma_\ell z_{j\ell} p_{c\ell} = 0$. It implies that multiplication of all prices by a common positive factor will have no effect. Only relative prices matter. It is known, therefore, as the *homogeneity property*. Taking both these properties into account means that instead of having to estimate $4 \times 4 = 16$ price coefficients, only 6 have to be determined – a considerable saving in degrees of freedom. A further property to be respected is the *negativity condition*, specialized here as $z_{ii} < 0$.

[1]In symbols,

$$V_t = \frac{[PIP(LI/100 + \delta)]_t}{[PIP(LI/100 + \delta)]_{1970}}$$

with *PIP*, the price index of non-residential fixed investments, *LI*, the long-run interest rate, measured as a percentage and $\delta$, the constant rate of depreciation which is set at 0.1.

The $z_{j\ell}$ have the dimension of a quantity concept divided by a price index. The quantity is expressed in currency units of 1970 and reflects both the scale and the currency of the economy in question. The $z_{j\ell}$ are, therefore, not easily comparable across countries. Rather than estimating the $z_{j\ell}$ themselves, we will estimate

$$\psi_{j\ell} = p_{cj,t-1}\, z_{j\ell t}\, p_{c\ell,t-1}/C_{t-1} \tag{35}$$

where $C_t$ is a type of aggregate 'cost' variable, defined[2] as

$$C_t = (WBU.NP/N + GOSH + IPO.V + QU + MGU)_t$$

It is easily seen that the $\psi_{j\ell}$ have no specific dimension and are comparable in value across countries. It is clear that symmetry in $j$ and $\ell$ is respected:

$$\psi_{j\ell} = \psi_{\ell j} \tag{36}$$

while, when interpreting the $z_{j\ell t}$ as partial derivatives evaluated for values of their arguments in $t-1$, the homogeneity property implies

$$\textstyle\sum_\ell \psi_{j\ell} = 0 \tag{37}$$

To see how the $\psi_{j\ell}$ are related to own- or cross-price elasticities note that

$$(\partial c_j/\partial p_{c\ell})_t = z_{j\ell t} = \psi_{j\ell}\, C_{t-1}/(p_{cj,t-1} \cdot p_{c\ell,t-1}) \tag{38}$$

Consequently, the price elasticity is given by

$$\begin{aligned}(\partial \ell n\, c_j/\partial\, \ell n\, p_{c\ell})_t &= p_{c\ell,t-1}\, z_{j\ell t}/c_{j,t-1}\\ &= \psi_{j\ell}\, C_{t-1}/(p_{cj,t-1} \cdot c_{j,t-1})\\ &= \psi_{j\ell}/w_{j,t-1}\end{aligned} \tag{39}$$

where $w_{jt} = (p_{cj}\, c_j/C)_t$, ie the share of input $j$ in total costs.

The elasticities are not constants but vary inversely with $w_{j,t-1}$. They do not satisfy as such the symmetry property.

The negativity condition implies that $\psi_{ii} < 0$. For the $\psi_{ij}$, $i \neq j$, a positive sign will indicate substitution, since the increase in the price of the other input ($j$) apparently causes a shift away from $j$ towards $i$. When $\psi_{ij}$ is negative, one has the case of complementarity of inputs $i$ and $j$.

The speed of adjustment parameters $\kappa_1$ and $\kappa_2$, which are positive but at most as large as one, provide no problem for international comparability. They will be used as such except for a change in notation, *viz* $\kappa_H$ for $\kappa_2$ and $\kappa_K$ for $\kappa_1$.

This choice of constants is next used to rewrite the input demand equations. Note that expression (38) for the $z_{j\ell}$ implies that

$$(z_{j1}/z_{11})_t = (\psi_{j1}/\psi_{11})\,(V_{t-1}/p_{cj,t-1})$$

$$(z_{j2}/z_{22})_t = (\psi_{j2}/\psi_{22})\,(PH_{t-1}/p_{cj,t-1})$$

Starting with input demand equation (34), we first divide both sides by

[2]In the definition of $C_t$, the aggregate cost variable, *WBU* denotes the wage bill, *GOSH* the operating surplus of households, *QU* the use of energy, *MQU*, the non-energy commodity imports, all measured in current prices. Non-residential fixed investment in constant prices, *IPO*, is multiplied by capital user cost, *V*, to represent capital cost. As such, *IPO.V* is not the cost of capital services, but also includes those of capital expansion. The wage bill, *WBU*, is multiplied by the ratio of non-government employment (*NP*) to total employment (*N*) to make it resemble the non-government wage costs. Those conventions also apply in Table 2.

$c_{j,t-1}$ and at the same time replace the $z_{j\ell}$. The result is

$$\frac{\Delta c_{jt}}{c_{j,t-1}} = \frac{\Delta q_{jt}}{c_{j,t-1}} + \sum_\ell \frac{\psi_{j\ell}}{w_{j,t-1}} \frac{\Delta p_{c\ell t}}{p_{c\ell,t-1}} + \frac{\psi_{j1}}{\psi_{11}} \frac{V_{t-1}}{w_{j,t-1}\,C_t} \frac{\kappa_{K^{-1}}}{\kappa_K} \Delta IPO_t + \frac{\psi_{j2}}{\psi_{22}} \frac{PH_{t-1}}{w_{j,t-1}\,C_t} \frac{\kappa_{H^{-1}}}{\kappa_H} \Delta^2\, HEN_t$$

Next we use the approximation

$$\Delta c_{jt}/c_{j,t-1} \sim \Delta\ \ell n\ c_{jt}$$

and analogous expressions for the other variables, while also

$$\Delta^2\ HEN_t/HEN_{t-1} \sim \Delta^2\ \ell n\ HEN_t$$

is being used. We can then write

$$\Delta\ \ell n\ c_{jt} = \Delta q_{jt}/c_{j,t-1} + [\textstyle\sum_\ell \psi_{j\ell}\, \Delta\ \ell n\ p_{c\ell t} + \frac{\psi_{j1}}{\psi_{11}} \frac{\kappa_{K^{-1}}}{\kappa_K} w_{K,t-1}\ \Delta\ \ell n\ IPO_t + \frac{\psi_{j2}}{\psi_{22}} \frac{\kappa_H{}^{-1}}{\kappa_H} w_{H,t-1}\ \Delta^2\ \ell n\ HEN_t]/w_{j,t-1} \tag{40}$$

It is recalled that $\Delta q_{jt}$ in the first term on the right-hand side is the finite variant of $dq_j$ defined in Equation (8). Therefore

$$\frac{\Delta q_{jt}}{c_{j,t-1}} = \frac{1}{c_{j,t-1}} \sum_i \beta_{ji}\ \Delta f_{it} \sim \frac{1}{c_{j,t-1}} \sum_i \beta_{ji}\ f_{i,t-1}\ \Delta \ell n\ f_{it} = \sum_i \beta_{ji} \frac{f_{i,t-1}}{c_{j,t-1}}\ \Delta \ell n\ f_{it}$$

with $f_{it}$ being a category of final demand (private consumption, government consumption of goods and services, and so on) while the $\beta_{ji}$ are the input contents – see the Appendix. The factor $f_{i,t-1}/c_{j,t-1}$ by which the $\beta_{ji}$ are multiplied is fixed at its value for $t - 1 = 1970$. Thus, $\Delta q_{jt}/c_{j,t-1}$ is approximately equal to

$$\Delta\ \ell n\ F_{jt} = \sum_i \beta_{ji} \frac{f_{i,1970}}{c_{j,1970}} \Delta\ \ell n\ f_{it}$$

Integrated versions of this concept are also being used:

$$\ell n\ F_{jt} = \sum_i \beta_{ji} \frac{f_{i,1970}}{c_{j,1970}}\ \ell n\ f_{it}$$

The input demand equations (40) are then written as

$$\begin{aligned}\Delta \ell n\ c_{jt} = {} & \Delta \ell n\ F_{jt} + [\Sigma_\ell\ \psi_{j\ell}\ \Delta\ell n\ p_{c\ell t} \\ & + \frac{\psi_{j1}}{\psi_{11}} \frac{\kappa_K - 1}{\kappa_K} w_{K,t-1}\ \Delta\ \ell n\ IPO_t \\ & + \frac{\psi_{j2}}{\psi_{22}} \frac{\kappa_H - 1}{\kappa_H} w_{H,t-1}\ \Delta^2\ \ell n\ HEN_t]/w_{j,t-1} \qquad (41)\end{aligned}$$

Expressions of the same type, *mutatis mutandis*, are valid when $\Delta\ \ell n\ HEN_t$ and $\Delta\ \ell n\ IPO_t$ are the dependent variables.

Until this far the production system was formulated without explicitly accounting for technical change. As a first approximation technological innovation can be represented under the form of an exponential trend. As far as our equations are in first differences of logarithms, which is the case when $\Delta\ell n\ HEN_t$, $\Delta\ell n\ QO_t$ and $\Delta\ell n\ MGO_t$ are dependent variables, the introduction of such a trend amounts to the insertion of an intercept $\alpha_{oj}$. A negative value for this constant means that technical change has resulted in saving on input $j$. It is, of course, possible that technical change saves on one input and uses more of another one.

The introduction of a technological trend in the equation for capital formation deserves special attention. The analogue of (41) for this input as derived from Equation (29) is

$$\begin{aligned}\Delta\ \ell n\ IPO_t = {} & \kappa_K\ [\Delta\ \ell n\ FIO_t - (1 - \delta)\Delta\ \ell n\ FIO_{t-1}] \\ & + \kappa_K\ [\Sigma_\ell\ \psi_{K\ell}\ (\Delta\ \ell n\ p_{c\ell t} - (1 - \delta)\Delta\ \ell n\ p_{c\ell,t-1})]/w_{K,t-1} \\ & + (1 - \kappa_K)\Delta\ \ell n\ IPO_{t-1} \qquad (42)\end{aligned}$$

Remembering that gross investment is close to being a first difference in capital stock and $\Delta\ \ell n\ IPO_t$ is a first difference in the logarithm of gross investment, Equation (42) is virtually in second differences in capital as a production factor. To bring it on the same footing as the other input equations, the equation is integrated over time on both sides. This involves an integration constant which is specified as

$$\alpha_{OK} + \alpha_{1K}\ t + \alpha_{2K}\ (\ell n\ DUC_{t-1} - (1 - \delta)\ \ell n\ DUC_{t-2})$$

to capture both technological change and the degree of utilization of capacity ($DUC$) which in the COMET context is an important adjustment modifying variable. The resulting equation reads

$$\begin{aligned}\ell n\ IPO_t = {} & \alpha_{OK} + \kappa_K\ [\ell n\ FIO_t - (1 - \delta)\ \ell n\ FIO_{t-1}] \\ & + \kappa_K\ [\Sigma_\ell\ \psi_{K\ell}\ (\ell n\ p_{c\ell t} - (1 - \delta)\ \ell n\ p_{c\ell,t-1})]/w_{K,t-1} \\ & + (1 - \kappa_K)\ell n\ IPO_{t-1} + \alpha_{1K} t + \alpha_{2K}\ (\ell n\ DUC_{t-1} \\ & - (1 - \delta)\ell n\ DUC_{t-2}) \qquad (43)\end{aligned}$$

After all these preliminaries we are now ready for an empirical application of these specifications.

## Estimation results

The system of equations derived in the preceding subsections is recapitulated as follows

$$\Delta\ \ell n\ HEN_t = \alpha_{0H} + \kappa_H\ (\ell n\ FNO_t - \ell n\ FNO_{t-2})/2$$
$$+\ \kappa_H\ \{\textstyle\sum_\ell \psi_{H\ell}\ \Delta\ \ell n\ p_{c\ell t} + (\psi_{HK}/\psi_{KK})\ [(\kappa_K - 1)/\kappa_K]\ WK_{t-1}\ \Delta \ell n$$
$$IPO_t\}/WH_{t-1} + (1-\kappa_H)\ \Delta\ \ell n\ HEN_{t-1} \qquad (44)$$

$$\Delta\ \ell n\ IPO_t = \alpha_{0K} + \kappa_K\ [\ell n\ FIO_t - (1-\delta)\ \ell n\ FIO_{t-1} - \ell n\ IPO_{t-1}]$$
$$+\ \kappa_K\ \{\textstyle\sum_\ell \psi_{K\ell}\ [\ell n\ p_{c\ell t} - (1-\delta)\ \ell n\ p_{c\ell,t-1}]\}/WK_{t-1}$$
$$+\ \alpha_{1K}\, t + \alpha_{2K}\ [\ell n\ DUC_{t-1} - (1-\delta)\ \ell n\ DUC_{t-2}] \qquad (45)$$

$$\Delta\ \ell n\ QO_t = \alpha_{0Q} + \Delta \ell n\ FQO_t$$
$$+\ \{\textstyle\sum_\ell \psi_{Q\ell}\ \Delta\ \ell n\ p_{c\ell t} + (\psi_{QK}/\psi_{KK})\ [(\kappa_K - 1)/\kappa_K] WK_{t-1}\ \Delta\ \ell n\ IPO_t$$
$$+\ (\psi_{QH}/\psi_{HH})\ [(\kappa_H - 1)/\kappa_H]\ WH_{t-1}\ \Delta^2\ \ell n\ HEN_t\}/WQ_{t-1} \qquad (46)$$

$$\Delta\ \ell n\ MGO_t = \alpha_{0M} + \Delta \ell n\ FMGO_t$$
$$+\ \{\textstyle\sum_\ell \psi_{M\ell}\ \Delta\ \ell n\ p_{c\ell t} + (\psi_{MK}/\psi_{KK})\ [(\kappa_K - 1)/\kappa_K] WK_{t-1}\ \Delta\ \ell n\ IPO_t$$
$$+\ (\psi_{MH}/\psi_{HH})\ [(\kappa_H - 1)/\kappa_H] WH_{t-1}\ \Delta^2 \ell n\ HEN_t\}/WM_{t-1} \qquad (47)$$

Table 2 facilitates tracing the correspondence between the various sets of symbols.

On the whole Equations (44)–(47) are simple transcriptions of Equations (41) and (43). The exception is the replacement of $\Delta\ \ell n\ FNO_t$ by

$$(\Delta\ \ell n\ FNO_t + \Delta\ \ell n\ FNO_{t-1})/2 = (\ell n\ FNO_t - \ell n\ FNO_{t-2}))/2$$

in Equation (44) to account for a smoothing of the labour requirement effect. Note that the use of $\Delta\ \ell n\ IPO_t$ as a dependent variable in Equation (45) rather than $\ell n\ IPO_t$ as in Equation (43) is the result of a simple rearrangement of terms.

In view of the symmetry and homogeneity conditions, the following constraints are imposed on the $\psi_{j\ell}$:

$$\psi_{KH} = \psi_{HK},\ \psi_{QH} = \psi_{HQ},\ \psi_{MH} = \psi_{HM} = -(\psi_{HH} + \psi_{HK} + \psi_{HQ})$$
$$\psi_{QK} = \psi_{KQ},\ \psi_{MK} = \psi_{KM} = -(\psi_{HK} + \psi_{KK} + \psi_{KQ})$$
$$\psi_{MQ} = \psi_{QM} = -(\psi_{HQ} + \psi_{KQ} + \psi_{QQ})$$
$$\psi_{MM} = \psi_{HH} + \psi_{KK} + \psi_{QQ} + 2(\psi_{HK} + \psi_{HQ} + \psi_{KQ})$$

The 'independent' $\psi_{j\ell}$ are $\psi_{HH}$, $\psi_{KK}$, $\psi_{QQ}$, $\psi_{HK}$, $\psi_{HQ}$ and $\psi_{KQ}$ which all appear in two or more of the equations. This calls for simultaneous estimation of Equations (44) to (47) together. Note that $\psi_{KK}$ occurs in a non-linear form in three of the four equations. This caused problems for

**Table 2.** Correspondence of symbols.

| $\ell$ | $P_{c\ell}$ | $F_\ell$ | $w_\ell$ |
|---|---|---|---|
| H | PH | FNO | WH : (WBU.NPIN + GOSH)/C |
| K | V | FIO | WK : (IPO.V)/C |
| Q | PQ | FQO | WQ : QU/C |
| M | PMG | FMGQ | WM : MGU/C |

estimation for 6 of the 13 country models. In those cases $\psi_{KK}$ was fixed at $-WK_{1970}$. This means that the own-price elasticity of capital input for 1971 was set at $-1$.

Also, $\kappa_H$ and $\kappa_K$ appear in all equations and in a nonlinear fashion. To keep estimation under control too, their values have been *a priori* set at 0.7 for $\kappa_H$ and 0.3 for $\kappa_K$. The lower value for the speed of adjustment for capital reflects the relative inertia (as compared to employment) to move to its equilibrium value. No attempt was made to estimate $\delta$, the rate of depreciation. It was set uniformly at 10% per year.

The remaining unknown coefficients have been estimated by a joint estimation procedure. The statistical feedback of the dependent variables on the variables on the right-hand side of the equations has been ignored. The resulting inconsistency is the price to be paid to benefit from the restrictions across equations while not complicating estimation too much. The restrictions will also act as a safety net to prevent excessive small sample bias. For most countries the sample period was 1955–1979. For HE and PO, however, 1955–1976 was used, while the results for ES are based on the data for 1957–1981.

Table 3 presents the estimation results. For each country, ie per block of four equations, 12 coefficients had to be determined. Their point estimates, with their estimated standard errors in parentheses below, are given row by row. For historical reasons the order of the countries is different from that in Table 1. As can be seen from Table 3, $\psi_{KK}$ had to be assigned an *a priori* value 6 times, $\psi_{QQ}$ and $\alpha_{2K}$ each once. The

**Table 3. Estimation results for coefficients factor demand module 1955–1979.**[a]

| Country | $\alpha_{0H}$ | $\alpha_{0K}$ | $\alpha_{0Q}$ | $\alpha_{0M}$ | $\psi_{HH}$ | $\psi_{HK}$ | $\psi_{KK}$ | $\psi_{HQ}$ | $\psi_{KQ}$ | $\psi_{QQ}$ | $\alpha_{1K}$ | $\alpha_{2K}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DB | −0.057 | 1.199 | −0.017 | 0.011 | −0.167 | 0.139 | −0.145 | 0.003 | 0.002 | −0.000 | 0.006 | 1.218 |
| | (0.041) | (0.044) | (0.053) | (0.053) | (0.152) | (0.036) | b | (0.025) | (0.014) | (0.010) | (0.006) | (1.819) |
| FR | −0.051 | 1.201 | −0.004 | −0.035 | −0.143 | 0.107 | −0.140 | 0.009 | 0.006 | −0.002 | 0.013 | 2.721 |
| | (0.040) | (0.048) | (0.067) | (0.080) | (0.103) | (0.037) | b | (0.020) | (0.010) | (0.006) | (0.006) | (3.191) |
| IT | −0.057 | 2.343 | 0.012 | 0.003 | −0.111 | 0.100 | −0.128 | 0.001 | 0.001 | −0.004 | 0.008 | 2.136 |
| | (0.041) | (0.046) | (0.049) | (0.054) | (0.153) | (0.136) | (0.174) | (0.006) | (0.007) | (0.007) | (0.006) | (2.066) |
| NL | −0.045 | 0.739 | 0.016 | −0.001 | −0.104 | 0.089 | −0.122 | 0.005 | 0.001 | −0.003 | 0.008 | 1.821 |
| | (0.045) | (0.047) | (0.061) | (0.073) | (0.211) | (0.066) | b | (0.036) | (0.015) | (0.018) | (0.006) | (1.876) |
| BE | −0.048 | 1.395 | −0.014 | −0.026 | −0.144 | 0.064 | −0.109 | 0.003 | −0.003 | −0.003 | 0.011 | 0.721 |
| | (0.048) | (0.049) | (0.060) | (0.096) | (0.293) | (0.088) | b | (0.032) | (0.018) | (0.010) | (0.066) | (1.987) |
| UK | −0.033 | 2.297 | 0.008 | −0.007 | −0.117 | 0.098 | 0.113 | −0.008 | −0.001 | −0.003 | 0.008 | 0.702 |
| | (0.041) | (0.046) | (0.056) | (0.069) | (0.129) | (0.071) | b | (0.029) | (0.025) | (0.013) | (0.006) | (2.225) |
| IR | −0.046 | 1.529 | 0.024 | 0.012 | −0.109 | 0.132 | −0.174 | −0.002 | −0.003 | −0.004 | 0.019 | 0.036 |
| | (0.041) | (0.045) | (0.049) | (0.050) | (0.200) | (0.159) | (0.199) | (0.020) | (0.011) | (0.008) | (0.006) | (1.696) |
| DK | −0.039 | 0.708 | 0.026 | −0.042 | −0.119 | 0.083 | −0.140 | −0.006 | −0.003 | −0.006 | 0.015 | 1.436 |
| | (0.043) | (0.047) | (0.072) | (0.084) | (0.225) | (0.157) | (0.301) | (0.023) | (0.014) | (0.008) | (0.006) | (2.041) |
| US | −0.020 | 1.203 | 0.020 | 0.019 | −0.133 | 0.127 | −0.134 | −0.003 | 0.005 | −0.003 | 0.010 | 1.049 |
| | (0.040) | (0.044) | (0.046) | (0.051) | (0.030) | (0.016) | b | (0.012) | (0.012) | (0.006) | (0.006) | (2.028) |
| JA | −0.088 | 2.541 | −0.003 | −0.027 | −0.160 | 0.178 | −0.195 | −0.008 | 0.004 | −0.000 | 0.024 | 2.758 |
| | (0.044) | (0.054) | (0.054) | (0.069) | (0.231) | (0.241) | (0.262) | (0.012) | (0.008) | (0.004) | (0.008) | (1.635) |
| HE | −0.056 | 0.898 | 0.064 | 0.001 | −0.137 | 0.134 | −0.170 | −0.014 | 0.008 | −0.001 | 0.026 | 0.200 |
| | (0.046) | (0.056) | (0.067) | (0.073) | (0.144) | (0.116) | (0.139) | (0.015) | (0.009) | (0.006) | (0.007) | b |
| PO | −0.053 | 0.820 | 0.024 | 0.029 | −0.154 | 0.153 | −0.169 | 0.008 | −0.007 | −0.001 | 0.014 | 0.788 |
| | (0.046) | (0.077) | (0.060) | (0.062) | (0.219) | (0.205) | (0.229) | (0.015) | (0.013) | b | (0.008) | (1.636) |
| ES | −0.049 | 1.521 | 0.086 | −0.055 | −0.182 | 0.118 | −0.126 | −0.010 | −0.002 | −0.001 | 0.021 | 1.780 |
| | (0.041) | (0.044) | (0.083) | (0.073) | (0.123) | (0.117) | (0.129) | (0.015) | (0.006) | (0.005) | (0.007) | (2.116) |
| Mean, standard deviation | −0.049 | 1.414 | 0.019 | −0.010 | −0.137 | 0.117 | −0.143 | −0.002 | 0.001 | −0.002 | 0.014 | 1.336 |
| | (0.015) | (0.598) | (0.028) | (0.025) | (0.023) | (0.030) | (0.025) | (0.007) | (0.004) | (0.002) | (0.006) | (0.840) |

[a] Sample period : *HE, PO* : 1955–1976; *ES* : 1957–1981.
[b] Assigned, ie not estimated, value.

bottom row of this table gives the unweighted means of the coefficient estimates across countries together with their standard deviation. With some exceptions, the point estimates are within a rather narrow range.

The values for $\psi_{HH}$, $\psi_{KK}$, $\psi_{QQ}$ and the implied ones for $\psi_{MM}$ are all negative to conform with the negativity condition. The $\psi_{HH}$, $\psi_{KK}$ and $\psi_{HK}$ are all relatively large in absolute value. Demand for labour and capital appears to be price sensitive. The substitution between capital and labour turns out to be surprisingly strong, witnessing the positive sign of $\psi_{HK}$. Energy and non-energy commodity demand have weak price responses. There is also not a clear pattern of substitution or complementarity between labour and these two input categories, between capital and energy and between energy and imports. The implied values for $\psi_{KM}$ are all positive but small, indicating a weak tendency for mutual substitutability between capital and imports. Own- and cross-price elasticities obtained by Equation (39) for 1980 are given in Table 5.

As is clear from Table 3, all countries display labour-saving technical change ($\alpha_{OH} < 0$). On average there is a trendlike loss of 5% in working hours annually. This holds for all non-government employment and not only for manufacturing. This type of labour saving is most outspoken for JA and weakest for US. The corresponding concept for capital is represented by the value of $\alpha_{1K}$. This coefficient is positive, and between 1% and 2% per year. Technical change appears to be capital using. The results for energy and non-energy commodity imports ($\alpha_{OQ}$, $\alpha_{OM}$) are mixed and weak. It is not easy to make firm statements about the general saving or using nature of technical change for these inputs.

With a few exceptions the effect of the degree of capacity utilization on investment activity has shown up. The standard errors are relatively large, however, and one cannot say that it is indispensable for the explanation of investment. What matters, though, is that the model contains the mechanism by which deviations from normal capacity utilization are corrected by investment activity.

As Table 4 shows, the standard errors of regression of the employment equation range from 2% for FR and UK to 7% for JA. This is not a very close fit. This quantity was less than 1% in the COMET II model (see Barten *et al* [1]). The difference is not only due to the fact that here the sample period contains data for the late 1970s with greater variability, but also to the fact that the employment equation is estimated together with the other equations of the module. It is not the

**Table 4. Statistical performance measures, factor demand module.**

| Country | Standard error of regression | | | | Durbin–Watson statistic | | | |
|---|---|---|---|---|---|---|---|---|
| | *H* | *K* | *Q* | *M* | *H* | *K* | *Q* | *M* |
| DB | 0.034 | 0.051 | 0.024 | 0.027 | 1.246 | 1.243 | 1.594 | 2.418 |
| FR | 0.020 | 0.033 | 0.036 | 0.049 | 1.690 | 1.248 | 2.673 | 2.191 |
| IT | 0.065 | 0.066 | 0.035 | 0.064 | 2.238 | 1.445 | 1.821 | 2.459 |
| NL | 0.030 | 0.062 | 0.061 | 0.027 | 1.612 | 1.940 | 1.805 | 2.404 |
| BE | 0.030 | 0.052 | 0.040 | 0.029 | 2.132 | 1.462 | 1.706 | 1.932 |
| UK | 0.020 | 0.034 | 0.034 | 0.042 | 2.124 | 1.804 | 1.406 | 2.302 |
| IR | 0.041 | 0.096 | 0.096 | 0.052 | 1.448 | 1.642 | 2.522 | 2.730 |
| DK | 0.027 | 0.047 | 0.057 | 0.037 | 1.932 | 1.927 | 2.009 | 2.620 |
| US | 0.027 | 0.057 | 0.025 | 0.059 | 2.017 | 1.139 | 1.710 | 2.299 |
| JA | 0.068 | 0.077 | 0.035 | 0.080 | 0.957 | 1.272 | 1.940 | 2.415 |
| HE | 0.036 | 0.139 | 0.086 | 0.073 | 2.125 | 1.596 | 2.550 | 2.198 |
| PO | 0.040 | 0.078 | 0.068 | 0.110 | 1.352 | 0.919 | 2.944 | 2.626 |
| ES | 0.030 | 0.085 | 0.063 | 0.088 | 1.676 | 1.016 | 1.539 | 1.415 |

standard error of regression of the employment equation which is minimized, but the determinant of the covariance matrix of the disturbances for all four equations. Here, price effects and dynamics of the various equations are intertwined. An improvement in fit for one equation can imply a reduction in fit for another one. A similar interconnection underlies the Durbin–Watson indicator of autocorrelation. By increasing the value $\kappa_K$ from 0.7 to 0.9 one would have moved the DW for investment up to values closer to 2 but at the expense of values of DW strongly above 2 for non-energy commodity imports. The

**Table 5.** Own- and cross-price elasticities of factor demand for 1980 and cost shares of 1979.

| Country | $i/j$ | $H$ | $K$ | $Q$ | $M$ | $W_{i,1979}$ |
|---|---|---|---|---|---|---|
| DB | $H$ | −0.256 | 0.213 | 0.004 | 0.038 | 0.655 |
| | $K$ | 1.161 | −1.206* | 0.015 | 0.029 | 0.120 |
| | $Q$ | 0.034 | 0.023 | −0.000 | 0.058 | 0.077 |
| | $M$ | 0.170 | 0.024 | −0.030 | −0.164 | 0.147 |
| FR | $H$ | −0.213 | 0.159 | −0.013 | 0.067 | 0.675 |
| | $K$ | 0.833 | −1.092* | 0.047 | 0.211 | 0.128 |
| | $Q$ | −0.199 | 0.135 | −0.045 | 0.110 | 0.045 |
| | $M$ | 0.293 | 0.178 | 0.035 | −0.509 | 0.152 |
| IT | $H$ | −0.173 | 0.157 | 0.002 | 0.014 | 0.641 |
| | $K$ | 0.885 | −1.129 | 0.013 | 0.232 | 0.113 |
| | $Q$ | 0.020 | 0.021 | −0.053 | 0.011 | 0.067 |
| | $M$ | 0.050 | 0.147 | 0.004 | −0.202 | 0.179 |
| NL | $H$ | −0.213 | 0.183 | 0.011 | 0.019 | 0.487 |
| | $K$ | 0.950 | −1.298* | 0.013 | 0.334 | 0.094 |
| | $Q$ | 0.035 | 0.008 | −0.017 | −0.026 | 0.149 |
| | $M$ | 0.034 | 0.116 | −0.014 | −0.136 | 0.270 |
| BE | $H$ | −0.276 | 0.122 | 0.005 | 0.149 | 0.523 |
| | $K$ | 0.643 | −1.098* | −0.028 | 0.483 | 0.099 |
| | $Q$ | 0.043 | −0.043 | −0.046 | 0.045 | 0.065 |
| | $M$ | 0.248 | 0.153 | 0.009 | −0.410 | 0.313 |
| UK | $H$ | −0.199 | 0.166 | −0.014 | 0.047 | 0.588 |
| | $K$ | 0.782 | −0.901* | −0.012 | 0.131 | 0.125 |
| | $Q$ | −0.075 | −0.013 | −0.031 | 0.119 | 0.111 |
| | $M$ | 0.155 | 0.936 | 0.075 | −0.324 | 0.176 |
| IR | $H$ | −0.239 | 0.289 | −0.005 | −0.044 | 0.456 |
| | $K$ | 0.938 | −1.237 | −0.024 | 0.323 | 0.140 |
| | $Q$ | −0.089 | −0.053 | −0.059 | 0.151 | 0.063 |
| | $M$ | −0.059 | 0.133 | −0.028 | −0.102 | 0.222 |
| DK | $H$ | −0.208 | 0.145 | −0.010 | 0.073 | 0.573 |
| | $K$ | 0.563 | −0.946 | −0.021 | 0.403 | 0.148 |
| | $Q$ | −0.099 | −0.053 | −0.113 | 0.265 | 0.057 |
| | $M$ | 0.188 | 0.269 | 0.068 | −0.525 | 0.222 |
| US | $H$ | −0.186 | 0.176 | −0.004 | 0.013 | 0.718 |
| | $K$ | 1.133 | −1.197* | 0.046 | 0.018 | 0.112 |
| | $Q$ | −0.024 | 0.045 | −0.024 | 0.003 | 0.115 |
| | $M$ | 0.173 | 0.036 | 0.007 | −0.215 | 0.055 |
| JA | $H$ | −0.223 | 0.249 | −0.011 | −0.014 | 0.718 |
| | $K$ | 1.041 | −1.136 | 0.023 | 0.072 | 0.171 |
| | $Q$ | −0.166 | 0.082 | −0.003 | 0.088 | 0.049 |
| | $M$ | −0.160 | 0.198 | 0.069 | −0.107 | 0.062 |
| HE | $H$ | −0.199 | 0.196 | −0.021 | 0.024 | 0.685 |
| | $K$ | 1.264 | −1.604 | 0.077 | 0.263 | 0.106 |
| | $Q$ | −0.229 | 0.130 | −0.017 | 0.116 | 0.063 |
| | $M$ | 0.133 | 0.192 | 0.050 | −0.355 | 0.146 |
| PO | $H$ | −0.237 | 0.236 | 0.012 | −0.011 | 0.652 |
| | $K$ | 1.282 | −1.419 | −0.055 | 0.192 | 0.120 |
| | $Q$ | 0.199 | −0.170 | −0.025* | −0.007 | 0.039 |
| | $M$ | −0.036 | 0.122 | −0.001 | −0.084 | 0.189 |
| ES | $H$ | −0.273 | 0.177 | −0.015 | 0.111 | 0.666 |
| | $K$ | 0.706 | −0.753 | −0.001 | 0.046 | 0.167 |
| | $Q$ | −0.136 | −0.003 | −0.015 | 0.153 | 0.072 |
| | $M$ | 0.184 | 0.082 | 0.118 | −0.983 | 0.095 |

*Based on preassigned value for $\psi_{ii}$.

present picture for DW is satisfactory on the whole. The tendency of positive autocorrelation for the investment equation is difficult to eliminate. Likewise, the standard errors of regression are, on the whole, not abnormally high for the type of activities explained. They are larger than one would have obtained with estimation of each equation separately, but this price for coherence and interpretability is willingly paid.

The elasticities of Table 5 give us an overall impression of the price effects per country. The entries are per row the elasticity of the demand for an input with respect to the price of labour ($H$), of capital ($K$), of energy ($Q$) and of non-energy commodity imports ($M$). The own-price elasticity for capital varies around unity. For six of the 13 cases this is so by construction. For DB and IT all inputs are substitutes, because all cross-price elasticities are positive. The apparent predominance of substitution is in part a formal matter. It follows from the homogeneity condition that there be at least one cross elasticity positive since the own-price elasticity is negative. This property is easily verified in Table 5. This table also shows that negative cross-price elasticities, ie complementarity, occur in all but two cases when energy is involved. As is to be expected, energy tends to be a complementary input. The own-price elasticity for energy is in absolute value the smallest among the own-price elasticities. The majority of cross-price elasticities are also close to zero. The general impression is that price sensitivity of input demand is not too strong.

## Conclusions

The results reported in the preceding section refer to a formulation of input demand functions which was the outcome of a series of steps of specifications, which were inspired by the desire to have an internally coherent subset of input equations, which display reasonably realistic dynamics, while keeping the number of parameters to be estimated low and maintaining a certain degree of comparability across countries. Coherence was obtained by starting from cost minimization with a single multi-output/multi-input production function and accounting for the spillover effects of incomplete adjustment. The two-step partial adjustment process has made it possible to introduce realistic dynamics without sacrificing coherence. The use of theoretical restrictions on the $\psi$ parameters and input–output information for the input contents of the various kinds of output has led to a relatively small number of parameters to be estimated. The chosen parameterization has resulted in point estimates which display surprisingly little variation across countries, which contributed to the objective of comparability. The price for these advantages is the reduced flexibility of each equation to fit the data of the sample. In the case of this module the advantages appear to be well worth this price.

## References

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# Appendix

## Primary input contents

This appendix explains how an input–output table is used to deduce the amount of each type of primary input needed to produce a unit of final demand output.

For a particular year an input–output table shows what each industry has used for primary inputs and products of other industries and how the product of each industry has been allocated over the other industries and their final uses (eg consumption, exports). For that year the following relation holds:

$$x = Ax + d \qquad \text{(A1)}$$

where $x$ is the vector of production levels for each of the $n$ industries and $d$ is the $n$-vector of final uses per industry. The $n \times n$ matrix $A$ is the matrix of 'technical coefficients' which can be deduced from the input–output table. If $I$-$A$ is a non-singular matrix, a not very restrictive assumption, Equation (A1) can also be written as:

$$x = (I - A)^{-1} d \qquad \text{(A2)}$$

which shows the production vector needed to satisfy a given vector of final uses.

From the input–output table one can also deduce the $q \times n$ matrix $C$ which shows how much primary inputs are needed per unit of output. Otherwise formulated:

$$c = Cx \qquad \text{(A3)}$$

where $c$ is the $q$-vector of primary inputs (eg labour, capital services). By definition, $\iota_n' A + \iota_q' C = \iota_n'$ where $\iota_n$ and $\iota_q$ are $n$- and $q$-vectors, respectively, of all elements equal to unity (summation vectors). Said otherwise, the elements of a column of $A$ together with the elements of the same column of $C$ add up to one. One can also write $\iota_q' C = \iota_n' (I\text{-}A)$.

Another piece of information which can be obtained from an input–output table is the allocation of the industry output for final uses over the different types of final demand like private and collective consumption, investment in fixed assets and stocks, exports. Let $f$ be the $p$-vector of the totals of final demand, then the $n \times p$ matrix $F$ satisfies the relation

$$d = Ff \qquad \text{(A4)}$$

Note that by definition the columns of $F$ add up to one, ie $\iota_n' F = \iota_p'$.

Combining Equations (A2), (A3) and (A4) one has:

$$c = C(I\text{-}A)^{-1} Ff = Bf \qquad \text{(A5)}$$

where $B$ is the $q \times p$ matrix $C(I\text{-}A)^{-1}F$. Its element $\beta_{ij}$ indicates how much of an input $i$ is needed to generate a unit of final demand $j$. This element is the desired primary input content.

A few remarks about $B$ are useful. Since matrices $C$, $(I\text{-}A)^{-1}$ and $F$ are all matrices with only non-negative elements also all elements of $B$, the $\beta_{ij}$, are not negative, and usually positive. Note also that the columns of $B$ add up to unity. Indeed one has:

$$\begin{aligned} \iota_q' B &= \iota_q' C(I\text{-}A)^{-1} F \\ &= \iota_n' (I\text{-}A)(I\text{-}A)^{-1} F \\ &= \iota_n' F = \iota_p' \end{aligned}$$

The matrix $B$, as it has been derived here, holds only for the year for which the input–output table is valid. However, as far as changes occur they will be gradual and minor. Even if for other years $B$ is not strictly valid, it still contains so much information not otherwise available that not to use it would be a waste. For this reason relation (A5) plays a role in the determination of factor demands.

The numerical derivation of $B$ is fairly straightforward if the appropriate input–output tables are available. The Statistical Office of the European Communities provided $B$ matrices for DB, IT, NL, BE, UK and DK for 1970. On the final demand side the following components are distinguished

| | |
|---|---|
| Household consumption | *CPU* |
| Government consumption of goods and services | *CGU* |
| Nonresidential investment | *IPU* |
| Investment in residential construction | *IRU* |
| Total stockbuilding | *STU* |
| Export of goods | *XGU* |
| Export of services | *XSU* |

followed by the symbols of the corresponding COMET variables in current prices. There are eight types of primary units, namely:

| | |
|---|---|
| Compensation of employees | *WBU* |
| Operating surplus | *GOST* |

**Table 6. Input contents, UK, 1970.**

| | *CPU* | *CGU* | *IPU* | *IRU* | *STU* | *XGU* | *XSU* |
|---|---|---|---|---|---|---|---|
| *H* | 0.570 | 0.821 | 0.551 | 0.735 | 0.336 | 0.611 | 0.480 |
| *K* | 0.102 | 0.032 | 0.076 | 0.079 | 0.070 | 0.087 | 0.126 |
| *NIT* | 0.148 | 0.074 | 0.065 | 0.080 | 0.158 | 0.111 | 0.061 |
| *MGU* | 0.143 | 0.047 | 0.277 | 0.076 | 0.375 | 0.145 | 0.033 |
| *MSU* | 0.017 | 0.019 | 0.021 | 0.009 | 0.012 | 0.021 | 0.262 |
| *MQU* | 0.019 | 0.008 | 0.010 | 0.019 | 0.047 | 0.025 | 0.039 |

| | |
|---|---|
| Consumption of fixed captial or total depreciation | *DPU* |
| Indirect taxes | *IT* |
| Subsidies (with a minus) | *SUB* |
| Imports of non-energy goods | *MGU* |
| Imports of services | *MSU* |
| Imports of total energy | *MQU* |

The factor demand module actually distinguishes only four inputs: labour, services of fixed assets, imports of non-energy goods and (imports of) total energy. Imports of services and taxes and subsidies are treated differently. Among the eight types of primary inputs, *WBU* and *DPU* are directly attributed to *H*, costs of labour, and to *K*, capital user costs, respectively. *GOST* represents in part labour income (eg of independents) and in part property income. Its row was assigned to that of *H* and *K* in accordance with the share of *GOSH*, the gross operating surplus of households, in *GOST*. This assumes that all non-wage income of households is (self-employed) labour income, clearly overstating the actual state of affairs, which, however, is not known. Since *GOSH* per output category is not available, the average share was used. Indirect taxes and subsidies are consolidated into $NIT = IT - SUB$. The corresponding row is obtained by subtracting the one for *SUB* from the one for *IT*. By way of illustration, the resulting 6 × 7 matix for UK is shown in Table 6.

The columns of this B matrix add up to one, apart from the presence of rounding errors. One may note the considerable variation of input contents per row. For example, *IPU* requires twice as much commodity imports as *XGU* or *CPU*, while *CGU* clearly shows its labour intensity.

For FR too a *B* table was supplied by *SOEC*, but the rows for *WBU*, *GOST*, *DPU*, *IT* and (minus)*SUB* were consolidated into a single one. This row was allocated to the *H*, *K*, *NIT* rows using the same proportion per column as that for DB.

For the other countries, namely IR, US, JA, HE, PO and ES, no *B* matrices were available with the same type of subdivision as that for the other countries. Consequently, these matrices were constructed using the RAS method (see Stone and Brown [3]). This method requires knowledge of the marginal totals of the matrix to be constructed. This was supplied by the relations already derived above, namely $\iota_q' B = \iota_p'$ and $c = Bf$. In the latter relation $c$ and $f$ were given the values of the input and output categories for 1970 for the country in question. In particular, $NIT = IT - SUB$, $H = WBU + GOSH$ and $K = YU - H - NIT$ was used next to the available data on *MGU*, *MSU* and *MQU*. The RAS method also needs a starting matrix. For this purpose a *B* matrix was used of another country which was believed to have a similar production structure. Thus, the *B* matrix of DB was used as starting matrix for US and JA, that of BE was used for IR, while the IT matrix served as starting matrix for HE, PO and ES.

All these calculations resulted in thirteen 6 × 7 tables of input contents like Table 6. It would take too much space to reproduce these here, but to make some comparisons Table 7 gives the non-energy commodity import and labour contents of commodity exports for all countries.

There appears to be considerable variation across countries. US and BE are extremes. US exports have the highest labour content and smallest import content, while for BE it is the other way around. Small economies do not necessarily have high import contents. In general, the labour contents are high when import contents are low and *vice versa*. One can expect an export impulse to generate quite a different reaction across countries.

The results of Table 7 in a way represent average rather than marginal contents. In a linear homogeneous framework like input–output analysis, there is no distinction between the two, but in practice one may expect a difference. The calculated input contents are, moreover, 1970 values, reflecting 1970 prices and 1970 technology. Since then, relative prices have changed (eg energy prices) and technology has also continued its labour saving trend. In spite of all these qualifications, the calculated *B* matrices can be taken to reflect at least the order of magnitude of the various input contents and are as such of precious empirical value.

**Table 7. Non-energy commodity import (*MGU*) and labour (*H*) input contents of commodity exports (*XGU*), 1970.**

| | *MGU* | *H* |
|---|---|---|
| DB | 0.156 | 0.615 |
| FR | 0.155 | 0.622 |
| IT | 0.163 | 0.623 |
| NL | 0.318 | 0.474 |
| BE | 0.395 | 0.430 |
| UK | 0.145 | 0.611 |
| IR | 0.341 | 0.464 |
| DK | 0.279 | 0.568 |
| US | 0.033 | 0.741 |
| JA | 0.053 | 0.617 |
| HE | 0.108 | 0.671 |
| PO | 0.169 | 0.597 |
| ES | 0.105 | 0.689 |